APPLICATION OF DERIVATIVES

BASIC CONCEPTS

1. **Tangents and Normals:**

   Equation of tangent to the curve $y = f(x)$ at the point (x_1, y_1) is given by

   $$(y - y_1) = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} (x - x_1)$$

   Equation of normal to the curve $y = f(x)$ at $P(x_1, y_1)$ is

   $$(y - y_1) = \frac{-1}{\left[\frac{dy}{dx}\right]_{(x_1, y_1)}} (x - x_1)$$

2. **Increasing Function:** A function $f(x)$ is said to be an increasing function in (a, b) if

   $$x_1 < x_2 \quad \Rightarrow \quad f(x_1) \le f(x_2) \quad \forall\, x_1, x_2 \in (a, b)$$

3. **Decreasing Function:** A function $f(x)$ is said to be decreasing in the interval (a, b) if

   $$x_1 < x_2 \quad \Rightarrow \quad f(x_1) \ge f(x_2) \quad \forall\, x_1, x_2 \in (a, b)$$

4. **Maximum and Minimum Value of a Function**

   (or Absolute Maximum or Minimum Value)

   A function f is said to attain maximum value at a point $a \in D_f$, if $f(a) \ge f(x) \;\; \forall\, x \in D_f$ *then* $f(a)$ is called absolute maximum value of f.

   A function f attains minimum value at $x = b \in D_f$, if $f(b) \le f(x) \;\; \forall\, x \in D_f$ *then* $f(b)$ is called absolute minimum value of f.

5. **Local Maxima and Local Minima (or Relative Extrema)**

   Local Maxima: A function $f(x)$ is said to attain a local maxima at $x = a$, if there exists a neighbourhood $(a - \delta, a + \delta)$ of 'a' such that $f(x) < f(a)\ \forall\, x \in (a - \delta, a + \delta)$, $x \neq a$, then $f(a)$ is the local maximum value of $f(x)$ at $x = a$.

   Local Minima: A function $f(x)$ is said to attain a local minima at $x = a$, if there exists a neighbourhood $(a - \delta, a + \delta)$ of 'a' such that $f(x) > f(a)\ \forall\, x \in (a - \delta, a + \delta)$, $x \neq a$, then $f(a)$ is called the local minimum value at $x = a$.

6. **Test for Identifying Relative (Local) Maxima or Minima**

   (*i*) **First Derivative Test**

   Step I: Find $f'(x)$

   Step II: The equation $f'(x) = 0$ is solved to get critical points $x = c_1, c_2, \ldots\ldots\ldots, c_n$.

   Step III: The sign of $f'(x)$ is studied in the neighbourhood of each critical points.

Let one critical point be $x = c$.

- If the sign of $f'(x)$ changes from +ve to –ve as x increases through c (from left to right of c).
  → $x = c$ is relative maxima and $f(c)$ is relative maximum value.
- If the sign of $f'(x)$ changes from –ve to +ve as x increases through c (from left to right of c).
  → $x = c$ is relative minima and $f(c)$ is relative minimum value.

(*ii*) **Second order derivative test**

Step I: Find $f'(x) = 0$.

Step II: The equation $f'(x) = 0$ is solved to get critical points $x = c_1, c_2, \ldots\ldots\ldots, c_n$.

Step III: $f''(x)$ is obtained and the sign of $f''(x)$ is studied for all critical points $x = c_1, c_2, \ldots\ldots\ldots, c_n$.

Let $x = c$ be one critical point.

- If $f''(c) > 0$ → $x = c$ is relative minima and $f(c)$ is relative minimum value.
- If $f''(c) < 0$ → $x = c$ is relative maxima and $f(c)$ is relative maximum value.
- If $f''(c) = 0$ → Second derivative test fail and first derivative test is used.

7. **Critical point:** A point $x = c$ is called critical point of the function $f(x)$, if $f(c)$ exists and either $f'(c) = 0$ or $f'(c) = \infty$ (does not exist).

8. **Point of Inflexion:** If $f(x)$ is a function and $x = c$ is critical point, then $x = c$ is called point of inflexion if

(*i*) $f'(c) = 0$ (*ii*) $f''(c) = 0$ (*iii*) $f'''(c) \neq 0$

MULTIPLE CHOICE QUESTIONS

Choose and write the correct option in the following questions.

1. **The interval in which the function f given by $f(x) = x^2 e^{-x}$ is strictly increasing, is** [*CBSE 2020 (65/2/1)*]

(*a*) $(-\infty, \infty)$ (*b*) $(-\infty, 0)$ (*c*) $(2, \infty)$ (*d*) $(0, 2)$

2. **The abscissa of the point on the curve $3y = 6x - 5x^3$, the normal at which passes through origin is**

(*a*) 1 (*b*) $\frac{1}{3}$ (*c*) 2 (*d*) $\frac{1}{2}$

3. **$f(x) = x^x$ has a stationary point at**

(*a*) $x = e$ (*b*) $x = \frac{1}{e}$ (*c*) $x = 1$ (*d*) $x = \sqrt{e}$

4. **The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 = 2$** [*NCERT Exemplar*]

(*a*) touch each other (*b*) cut at right angle

(*c*) cut at an angle $\frac{\pi}{3}$ (*d*) cut at an angle $\frac{\pi}{4}$

5. **The slope of normal to the curve $y = 2x^2 + 3 \sin x$ at $x = 0$ is**

(*a*) 3 (*b*) $\frac{1}{3}$ (*c*) –3 (*d*) $-\frac{1}{3}$

6. **The equation of the normal to the curve $y = \sin x$ at (0, 0) is**

(*a*) $x = 0$ (*b*) $y = 0$ (*c*) $x + y = 0$ (*d*) $x - y = 0$

7. **The tangent to the curve $y = e^{2x}$ at the point (0, 1) meets x-axis at**

(*a*) (0, 1) (*b*) (0, 2) (*c*) $\left(-\frac{1}{2}, 0\right)$ (*d*) (2, 0)

8. **The line $y = x + 1$ is a tangent to the curve $y^2 = 4x$ at the point**

(*a*) (1, 2) (*b*) (2, 1) (*c*) (1, – 2) (*d*) (–1, 2)

9. **The point on the curve $x^2 = 2y$ which is nearest to the point (0, 5) is**

(*a*) $(2\sqrt{2}, 4)$ (*b*) $(2\sqrt{2}, 0)$ (*c*) (0, 0) (*d*) (2, 2)

10. **The slope of tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at the point (2, –1) is**

(*a*) $\frac{22}{7}$ (*b*) $\frac{6}{7}$ (*c*) $\frac{7}{6}$ (*d*) $\frac{-6}{7}$

11. **The greatest of the numbers $1, 2^{1/2}, 3^{1/3}, 4^{1/4}, 5^{1/5}, 6^{1/6}$ and $7^{1/7}$, is**

(*a*) $2^{1/2}$ (*b*) $3^{1/3}$ (*c*) $4^{1/4}$ (*d*) $7^{1/7}$

12. **The minimum value of x^x ($x > 0$) is**

(*a*) 1 (*b*) $e^{-1/e}$ (*c*) $\left(\frac{1}{e}\right)^e$ (*d*) none of these

13. **The values of a for which $y = x^2 + ax + 25$ touches the x-axis are**

(*a*) 0 (*b*) ±10 (*c*) 4, – 6 (*d*) ± 5

14. **If $f(x) = \frac{1}{4x^2 + 2x + 1}$, then its maximum value is**

(*a* 0 (*b*) $\frac{4}{3}$

(*c*) ±5 (*d*) Maximum value does not exist.

15. **The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in**

(*a*) $\left(0, \frac{\pi}{2}\right)$ (*b*) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$ (*c*) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (*d*) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

16. **Let the $f : R \to R$ be defined by $f(x) = 2x + \cos x$, then f**

(*a*) has a maximum, at $x = 0$ (*b*) has a minimum, at $x = \pi$

(*c*) is an increasing function (*d*) is a decreasing function

17. **At $x = \frac{5\pi}{6}$, $f(x) = 2 \sin 3x + 3 \cos 3x$ is**

(*a*) 0 (*b*) minimum (*c*) maximum (*d*) none of these

18. **At (0, 0) the curve $y = x^{1/2}$ has a**

(*a*) a horizontal tangent parallel to x-axis (*b*) a vertical tangent parallel to y-axis

(*c*) an oblique tangent (*d*) tangent does not exist.

19. **If the tangent at (a, b) to the curve $x^3 + y^3 = c^3$ meets the curve again in (a_1, b_1) then $\frac{a_1}{a} + \frac{b_1}{b}$ is equal to**

(*a*) 1 (*b*) –1 (*c*) c (*d*) $\frac{c}{2}$

20. **The point of intersection of the tangent drawn to the curve $x^2y = 1 - y$ at the points where it is met by the curve $xy = 1 - y$, is given by**

(*a*) (0, – 1) (*b*) (1, 1) (*c*) (0, 1) (*d*) (–1, 0)

21. **The values of a for which the function $f(x) = \sin x - ax + b$ increases on R are**

(*a*) $(-\infty, \infty)$ (*b*) $[-1, 1]$ (*c*) $(-\infty, -1)$ (*d*) none of these

22. **The function $f(x) = \frac{2x^2 - 1}{x^4}$, $x > 0$, decreases in the interval**

(*a*) $(-\infty, 0)$ (*b*) $[1, \infty)$ (*c*) $[-1, -1]$ (*d*) none of these

23. The least value of the function $f(x) = ax + \frac{b}{x} (a > 0, b > 0, x > 0)$ is

(a) $\frac{a}{b}$ (b) $2\sqrt{ab}$ (c) 0 (d) none of these

24. The condition that the curve $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ may cut each other orthogonally is

(a) $a_1 + a = b_1 + b$ (b) $a_1 - a = b_1 - b$

(c) $\frac{1}{a_1} - \frac{1}{a} = \frac{1}{b_1} - \frac{1}{b}$ (d) $\frac{1}{a_1} + \frac{1}{a} = \frac{1}{b_1} + \frac{1}{b}$

25. If m be the slope of a tangent to the curve $e^y = 1 + x^2$ then

(a) $|m| > 1$ (b) $m < 1$ (c) $|m| < 1$ (d) $|m| \leq 1$

26. If at each point of the curve $y = x^3 - ax^2 + x + 1$ the tangent is inclined at an acute angle with positive direction of the x-axis then

(a) $a > 0$ (b) $a \leq \sqrt{3}$ (c) $-\sqrt{3} \leq a \leq \sqrt{3}$ (d) none of these

27. Slope of tangent to the curve $y = x^2 - x$ at the point where the line $y = 2$ cuts the curve in the first quadrant is

(a) 2 (b) 3 (c) -3 (d) None of these

28. The slope of the tangent of the locus $y = \cos^{-1}(\cos x)$ at $x = -\frac{\pi}{4}$ is

(a) 1 (b) 0 (c) 2 (d) -1

29. Let the parabolas $y = x^2 + ax + b$ and $y = x(c - x)$ touch each other at the point (1, 0). Then

(a) $a = -3$ (b) $b = 1$ (c) $c = 2$ (d) $b = -2$

30. Let $y = f(x)$ be the equation of parabola having its axis parallel to y axis, which is touched by the line $y = x$ at the point where $x = 1$. Then

(a) $f'(0) = f'(1)$ (b) $f'(1) = 0$

(c) $f(0) + f'(0) + f''(0) = 1$ (d) $2f(0) = 1 - f'(0)$

31. A point on the ellipse $4x^2 + 9y^2 = 36$ where the tangent is equally inclined to the axis is

(a) $\left(\frac{9}{\sqrt{13}}, \frac{4}{\sqrt{13}}\right)$ (b) $\left(-\frac{9}{\sqrt{13}}, \frac{4}{\sqrt{13}}\right)$ (c) $\left(\frac{9}{\sqrt{13}}, -\frac{4}{\sqrt{13}}\right)$ (d) All of these

32. If $xy = a^2$ and $S = b^2x + c^2y$ where a, b and c are positive constants then the minimum value of S is

(a) abc (b) $bc\sqrt{a}$ (c) $2abc$ (d) none of these

33. The global minimum value of $f(x) = x^4 - x^2 - 2x + 6$ is

(a) 6 (b) 8

(c) 4 (d) does not exists

34. The minimum value of $f(x) = 3\cos^2 x + 4\sin^2 x + \cos\frac{x}{2} + \sin\frac{x}{2}$ is

(a) 4 (b) $3 + \sqrt{2}$ (c) $4 + \sqrt{2}$ (d) none of these

35. If $a > b > 0$, the minimum value of $a \sec\theta - b\tan\theta$ is

(a) $b - a$ (b) $\sqrt{a^2 + b^2}$ (c) $\sqrt{a^2 - b^2}$ (d) $2\sqrt{a^2 - b^2}$

36. The number of values of x where the function $f(x) = \cos x + \cos(\sqrt{2}x)$ attains its maximum is

(a) 0 (b) 1 (c) 2 (d) infinite

37. If $\theta + \phi = \frac{\pi}{3}$ then $\sin\theta\sin\phi$ has a maximum value if θ is

(a) $\frac{\pi}{6}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{4}$ (d) none of these

38. If $f(x) = a \log_e |x| + bx^2 + x$ has extremum at $x = 1$ and $x = 3$ then

(*a*) $a = -\frac{3}{4}, b = -\frac{1}{8}$ (*b*) $a = \frac{3}{4}, b = -\frac{1}{8}$ (*c*) $a = -\frac{3}{4}, b = \frac{1}{8}$ (*d*) None of these

39. Let $f(x)$ be a function such that $f'(a) \neq 0$. Then at $x = a$, $f(x)$

(*a*) cannot have a maximum

(*b*) cannot have a minimum

(*c*) must have neither a maximum nor a minimum

(*d*) None of these

40. The function $f(x) = \sin^4 x + \cos^4 x$ increases if

(*a*) $0 < x < \frac{\pi}{8}$ (*b*) $\frac{\pi}{4} < x < \frac{3\pi}{8}$ (*c*) $\frac{3\pi}{4} < x < \frac{5\pi}{8}$ (*d*) $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

41. If $f(x) = \frac{x}{\sin x}$ and $g(x) = \frac{x}{\tan x}$, where $0 < x \leq 1$, then in the interval

(*a*) both $f(x)$ and $g(x)$ are increasing functions

(*b*) both $f(x)$ and $g(x)$ are decreasing functions

(*c*) $f(x)$ is an increasing function.

(*d*) $g(x)$ is an increasing function.

42. Let $h(x) = f(x) - \{f(x)\}^2 + \{f(x)\}^3$ for all real values of x. Then

(*a*) h is increasing whenever $f(x)$ is increasing

(*b*) h is increasing whenever $f'(x) < 0$

(*c*) h is decreasing whenever f is increasing

(*d*) nothing can be said in general

43. If $f(x) = \sin x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, then

(*a*) $f(x)$ is increasing in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(*b*) $f\{f(x)\}$ is increasing in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(*c*) $\left[f(f(x)) \text{ is invertible in} \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

(*d*) All of these

44. Let $f(x) = 2\sin^3 x - 3\sin^2 x + 12\sin x + 5$, $0 \leq x \leq \frac{\pi}{2}$. Then $f(x)$ is

(*a*) decreasing in $\left[0, \frac{\pi}{2}\right]$

(*b*) increasing in $\left[0, \frac{\pi}{2}\right]$

(*c*) increasing in $\left[0, \frac{\pi}{4}\right]$ and decreasing in $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

(*d*) none of these

45. Which of the following function is decreasing on $\left(0, \frac{\pi}{2}\right)$?

(*a*) $\tan 2x$ (*b*) $\cos x$ (*c*) $\cos 3x$ (*d*) none of these

46. The tangent to the curve given by $x = e^t . \cos t$, $y = e^t . \sin t$ at $t = \frac{\pi}{4}$ makes with x-axis an angle:

(*a*) 0 (*b*) $\frac{\pi}{4}$ (*c*) $\frac{\pi}{3}$ (*d*) $\frac{\pi}{2}$

47. The equation of normal to the curve $3x^2 - y^2 = 8$ which is parallel to the line $x + 3y = 8$, is

(*a*) $3x - y = 8$ (*b*) $3x + y + 8 = 0$ (*c*) $x + 3y \pm 8 = 0$ (*d*) $x + 3y = 0$

48. The equation of tangent to the curve $y(1 + x^2) = 2 - x$, where it crosses x-axis, is

(*a*) $x + 5y = 2$ (*b*) $x - 5y = 2$ (*c*) $5x - y = 2$ (*d*) $5x + y = 2$

49. The interval on which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing is

(*a*) $[-1, \infty)$ (*b*) $[-2, -1]$ (*c*) $(-\infty, -2]$ (*d*) $[-1, 1]$

50. The function $f(x) = 4 \sin^3 x - 6 \sin^2 x + 12 \sin x + 100$ is strictly

(*a*) Increasing in $\left(\pi, \frac{3\pi}{2}\right)$ (*b*) decreasing in $\left(\frac{\pi}{2}, \pi\right)$

(*c*) decreasing in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (*d*) decreasing in $\left(0, \frac{\pi}{2}\right)$

51. The function $f(x) = \tan x - x$

(*a*) always increases (*b*) always decreases

(*c*) never increases

(*d*) sometimes increases and sometimes decreases

52. If x is real, then the minimum value of $x^2 - 8x + 17$ is

(*a*) –1 (*b*) 0 (*c*) 1 (*d*) 2

53. The smallest value of polynomial $x^3 - 18x^2 + 96x$ in [0, 9] is

(*a*) 126 (*b*) 0 (*c*) 135 (*d*) 160

54. The function $f(x) = 2x^3 - 3x^2 - 12x + 4$, has

(*a*) two points of local maximum (*b*) two points of local minimum

(*c*) one maxima and one minima (*d*) no maxima or minima

55. The maximum value of $\sin x \,.\, \cos x$ is

(*a*) $\frac{1}{4}$ (*b*) $\frac{1}{2}$ (*c*) $\sqrt{2}$ (*d*) $2\sqrt{2}$

56. The maximum slope of curve $y = -x^3 + 3x^2 + 9x - 27$ is

(*a*) 0 (*b*) 12 (*c*) 16 (*d*) 32

57. If $x + y = 8$ then the maximum value of xy is

(*a*) 8 (*b*) 16 (*c*) 20 (*d*) 24

58. Find the slope of normal line to the curve: $x^2 - xy + 3y^2 - 5y = 0$ at $x = 2$.

(*a*) $\left(\frac{13}{8}, 3\right)$ (*b*) $\left(\frac{3}{8}, \frac{-1}{3}\right)$ (*c*) $\left(\frac{-8}{3}, \frac{1}{3}\right)$ (*d*) $\left(\frac{8}{3}, 6\right)$

59. Find the point at which the normal to the curve: $y = 2x^2 - 2x + 7$ has a slope $\frac{1}{6}$.

(*a*) (–1, –11) (*b*) (1, –11) (*c*) (–1, 11) (*d*) (–1, –9)

60. Find the angle of intersection of the two curves $x^2y = 2$ and $xy^2 = 4$.

(*a*) $\tan^{-1}\frac{3}{5}$ (*b*) $\tan^{-1}3$ (*c*) $\tan^{-1}\frac{5}{3}$ (*d*) None of these

61. It is given that for the function f given by $f(x) = x^3 + bx^2 + ax$, $x \in [1, 3]$, then

(*a*) $a = 11, b = -6$ (*b*) $a = -6, b = 11$ (*c*) $a = 6, b = 11$ (*d*) None of these

62. The slope of the curve $2y^2 = ax^2 + b$ at (1, –1) is –1. Find a, b.

(*a*) $a = 2, b = 0$ (*b*) $a = 2, b = 1$ (*c*) $a = 0, b = 2$ (*d*) $a = 1, b = 2$

63. **The angle between the curve $y^2 = x$ and $x^2 = y$ at (1, 1) is**

(*a*) 90° (*b*) $\tan^{-1}\frac{3}{4}$ (*c*) $\tan^{-1}\frac{4}{3}$ (*d*) 45°

64. **Find the maximum and minimum values of $f(x) = x + \sin 2x$ in the interval $[0, 2\pi]$.**

(*a*) Maximum value = 2π
Minimum value = 0

(*b*) Maximum value = 0
Minimum value = 2π

(*c*) Maximum value = 0
Minimum value = 0

(*d*) None of these

65. **If the curve $a\,y + x^2 = 7$ and $x^3 = y$, cut orthogonally at (1, 1), then the value of a is**

(*a*) 1 (*b*) 0 (*c*) –6 (*d*) 6

66. **The total revenue received from the sale of x units of a product is given by: $R(x) = 5x^3 - 4x^2$, find the marginal revenue of $x = 20$.**

(*a*) 5860 (*b*) 5840 (*c*) 5000 (*d*) 5600

67. **Find the equations of a tangent to the curve $x^3 - 2x^2y + xy^2 = 1$ at (1, – 1).**

(*a*) $2x - y = 3$ (*b*) $2x = 3$ (*c*) $2x + y = 3$ (*d*) $2x + y = -3$

68. **The point on the curve $y = 12x - x^2$ where the slope of the tangent is zero will be**

(*a*) (3, 9) (*b*) (2, 16) (*c*) (6, 36) (*d*) None of these

69. **Find the intervals in which $f(x) = -x^2 - 2x + 15$ is increasing or decreasing**

(*a*) Increasing $(-\infty, -1)$
Decreasing $(-1, \infty)$

(*b*) Increasing $(\infty, -2)$
Decreasing $(0, \infty)$

(*c*) Increasing $(-\infty, -4)$
Decreasing $(-4, \infty)$

(*d*) None of these

70. **The maximum value of slope of the curve $y = -x^3 + 3x^2 + 12x - 5$ is**

(*a*) 15 (*b*) 12 (*c*) 9 (*d*) 0

71. **It is given that at $x = 1$, the function $f(x) = x^4 - 62x^2 + ax + a$ attains its maximum value, on the interval [0, 2]. The value of a is**

(*a*) 20 (*b*) –120 (*c*) 120 (*d*) 52

Answers

1. (*d*)	**2.** (*a*)	**3.** (*b*)	**4.** (*b*)	**5.** (*d*)	**6.** (*c*)
7. (*c*)	**8.** (*a*)	**9.** (*a*)	**10.** (*b*)	**11.** (*b*)	**12.** (*b*)
13. (*b*)	**14.** (*b*)	**15.** (*b*)	**16.** (*c*)	**17.** (*b*)	**18.** (*b*)
19. (*b*)	**20.** (*c*)	**21.** (*c*)	**22.** (*b*)	**23.** (*b*)	**24.** (*c*)
25. (*d*)	**26.** (*c*)	**27.** (*b*)	**28.** (*a*)	**29.** (*a*)	**30.** (*d*)
31. (*d*)	**32.** (*c*)	**33.** (*c*)	**34.** (*c*)	**35.** (*c*)	**36.** (*b*)
37. (*a*)	**38.** (*a*)	**39.** (*d*)	**40.** (*b*)	**41.** (*c*)	**42.** (*a*)
43. (*d*)	**44.** (*b*)	**45.** (*b*)	**46.** (*d*)	**47.** (*c*)	**48.** (*a*)
49. (*b*)	**50.** (*b*)	**51.** (*a*)	**52.** (*c*)	**53.** (*b*)	**54.** (*c*)
55. (*b*)	**56.** (*b*)	**57.** (*b*)	**58.** (*b*)	**59.** (*c*)	**60.** (*a*)
61. (*a*)	**62.** (*a*)	**63.** (*b*)	**64.** (*a*)	**65.** (*d*)	**66.** (*b*)
67. (*a*)	**68.** (*c*)	**69.** (*a*)	**70.** (*a*)	**71.** (*c*)	

CASE-BASED QUESTIONS

Choose the correct option in the following questions.

1. Read the following and answer any four questions from (*i*) to (*v*).

The Relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$ where x is the number of days exposed to sunlight.

Based on the above informations answer the following:

(*i*) The rate of growth of the plant with respect to sunlight is

(*a*) $4x - \frac{1}{2}x^2$ (*b*) $4 - x$ (*c*) $x - 4$ (*d*) $x - \frac{1}{2}x^2$

(*ii*) What is the number of days it will take for the plant to grow to the maximum height?

(*a*) 4 (*b*) 6 (*c*) 7 (*d*) 10

(*iii*) What is the maximum height of the plant?

(*a*) 12 cm (*b*) 10 cm (*c*) 8 cm (*d*) 6 cm

(*iv*) What will be the height of the plant after 2 days?

(*a*) 4 cm (*b*) 6 cm (*c*) 8 cm (*d*) 10 cm

(*v*) If the height of the plant is $\frac{7}{2}$ cm, the number of days it has been exposed to the sunlight is

(*a*) 2 cm (*b*) 3 cm (*c*) 4 cm (*d*) 1 cm

Sol. (*i*) We have,

the rate of growth $= \frac{dy}{dx}$

$$= \frac{d\left(4x - \frac{1}{2}x^2\right)}{dx}$$

$= 4 - x$

$\therefore$ Option (*b*) is correct.

(*ii*) For height to be maximum or minimum

$$\frac{dy}{dx} = 0 \Rightarrow 4 - x = 0 \Rightarrow x = 4$$

$$\therefore \frac{d^2y}{dx^2} = -1 < 0 \Rightarrow y \text{ will be maximum when } x = 4$$

$\therefore$ Number of required days = 4

$\therefore$ Option (*a*) is correct.

(*iii*) We have, $y = 4x - \frac{1}{2}x^2$

$\therefore$ When $x = 4$ the height of the plant will be maximum which is

$$y = 4 \times 4 - \frac{1}{2} \times (4)^2 = 16 - 8 = 8 \text{ cm}$$

$\therefore$ Option (*c*) is correct.

(*iv*) Height of the plant after 2 days is given by

$$y = 4 \times 2 - \frac{1}{2} \times (2)^2 = 8 - 2 = 6 \text{ cm}$$

$\therefore$ Option (*b*) is correct.

(*v*) Given height of the plant, $y = \frac{7}{2}$

$\therefore \quad \frac{7}{2} = 4x - \frac{1}{2}x^2 \Rightarrow 7 = 8x - x^2$

$\Rightarrow \quad x^2 - 8x + 7 = 0 \Rightarrow x^2 - 7x - x + 7 = 0$

$\Rightarrow x(x-7) - 1(x-7) = 0$

$\Rightarrow (x-1)(x-7) = 0$

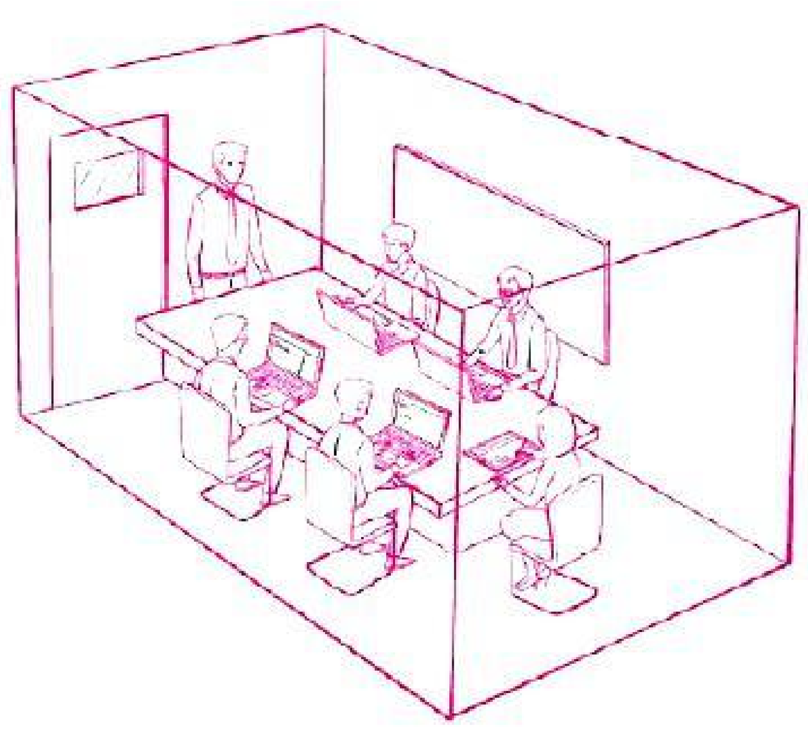
$\Rightarrow x = 1 \text{ or } x = 7$

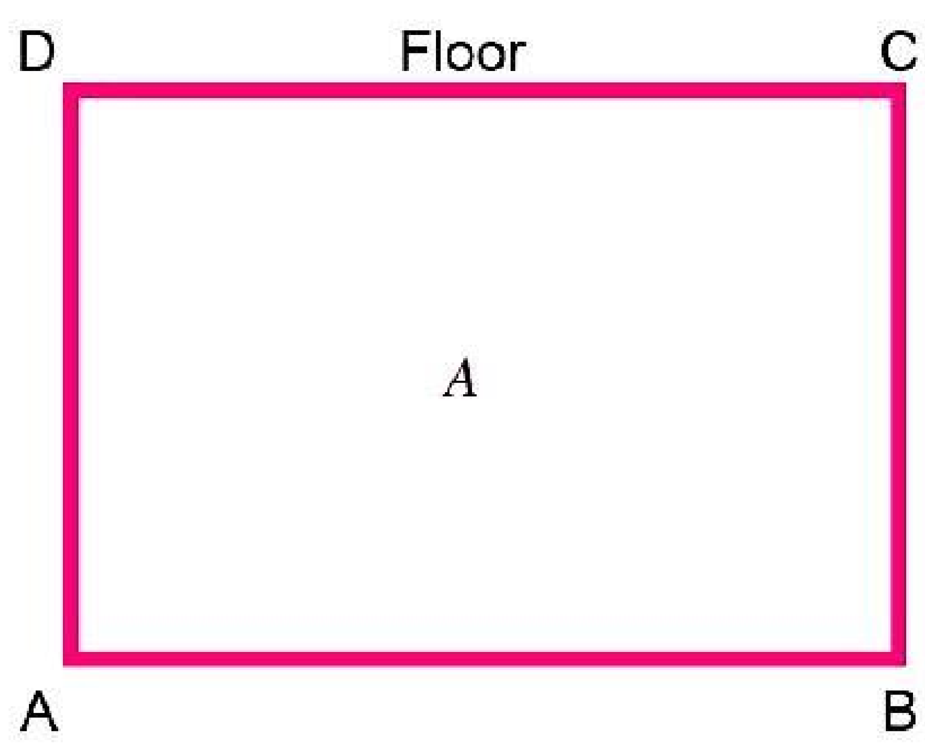
$x \neq 7$

$\therefore \quad x = 1 \text{ cm}$

$\therefore$ Option (*d*) is correct.

2. A rectangular hall is to be developed for a meeting of farmers in an agriculture college to aware them for new technique in cultivation. It is given that the floor has a fixed perimeter P as shown below.





Based on the above information answer the following.

(*i*) If x and y represents the length and breadth of the rectangular region, then the relation between the variables is

(*a*) $x + y = P$ (*b*) $x^2 + y^2 = P$ (*c*) $2(x + y) = P$ (*d*) $x + 2y = P$

(*ii*) The area of the rectangular region '*A*' expressed as a function of x is

(*a*) $\frac{1}{2}(P + x^2)$ (*b*) $\frac{1}{2}(Px - 2x^2)$ (*c*) $\frac{1}{2}(Px + 2x^2)$ (*d*) $Px - 2x^2$

(*iii*) Principal of agriculture college is interested in maximizing the area of floor '*A*'. For this to happen the value of x should be

(*a*) P (*b*) $\frac{P}{2}$ (*c*) $\frac{2P}{3}$ (*d*) $\frac{P}{4}$

(*iv*) To maximizing the area of floor '*A*'. For this to happen the value of y should be

(*a*) $\frac{P}{2}$ (*b*) $\frac{P}{4}$ (*c*) $\frac{P}{3}$ (*d*) $\frac{2P}{5}$

(*v*) **The maximum value of area of floor '*A*' is**

(*a*) $\frac{P^2}{8}$ (*b*) $\frac{2P}{9}$ (*c*) $\frac{P}{10}$ (*d*) $\frac{P^2}{16}$

Sol. (*i*) Perimeter of rectangle $ABCD = 2(l + b) = 2(x + y)$

Option (*c*) is correct.

(*ii*) $\because A = x.y$

$= x.\frac{P-2x}{2}$

$= \frac{Px - 2x^2}{2}$

$$\left[\begin{array}{l} \text{Given} \\ \text{Perimeter} = P \\ \Rightarrow 2(x+y) = P \\ \Rightarrow y = \frac{P}{2} - x \\ \quad = \frac{P-2x}{2} \end{array}\right]$$

Option (*b*) is correct.

(*iii*) $\because \quad A = \frac{Px - 2x^2}{2}$

$\Rightarrow \frac{dA}{dx} = \frac{P-4x}{2}$

For maximum or minimum value of x

$$\frac{dA}{dx} = 0 \Rightarrow \frac{P-4x}{2} = 0$$

$\Rightarrow P - 4x = 0 \Rightarrow x = \frac{P}{4}$

Also, $\left.\frac{d^2A}{dx^2}\right]_{x=\frac{P}{4}} = -2\,(-\text{ve})$

Option (*d*) is correct.

(*iv*) Putting $x = \frac{P}{4}$ in $y = \frac{P-2x}{2}$

$\Rightarrow y = \frac{P - 2\times\frac{P}{4}}{2}$

$= \frac{4P-2P}{8} = \frac{2P}{8} = \frac{P}{4}.$

Option (*b*) is correct.

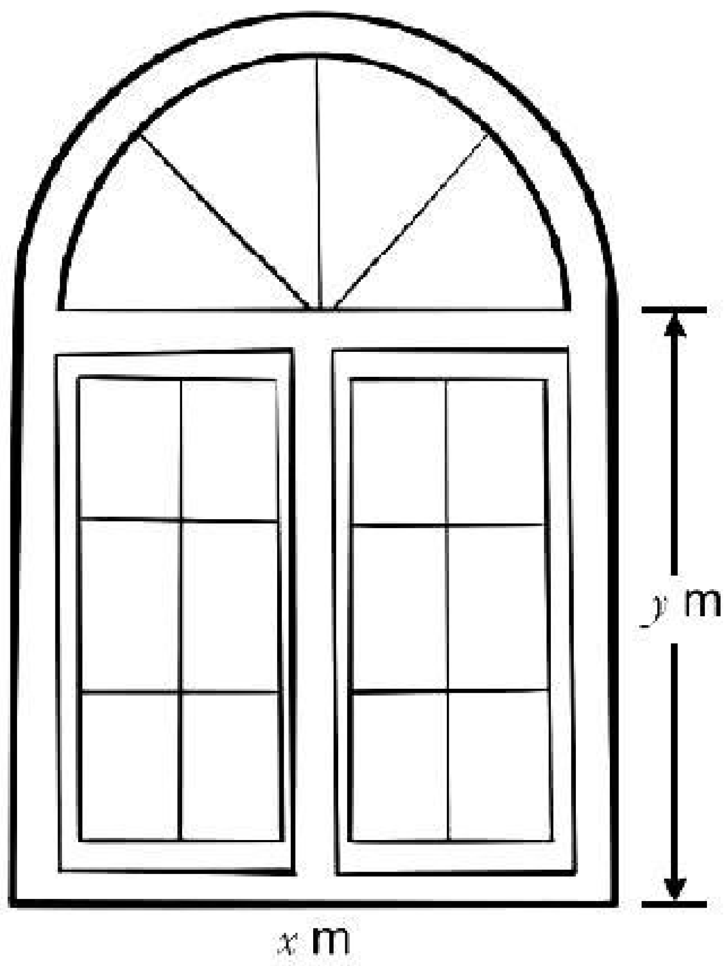
(*v*) Since A = length × breadth

$= x \times y$

$= \frac{P}{4} \times \frac{P}{4} = \frac{P^2}{16}.$

Option (*d*) is correct.

3. Dr. Ritam residing in Delhi went to see an apartment of 3 BHK in Noida. The window of the house was in the form of a rectangle surmounted by a semicircular opening having a perimeter of the window 10 m as shown in figure.



Based on above information answer the following.

(i) If x any y represents the length and breadth of the rectangular region, then the relation between the variables is

(a) $x + y + \frac{x}{2} = 10$ (b) $x + 2y + \frac{x}{2} = 10$ (c) $x + 2y + \pi\frac{x}{2} = 10$ (d) $2x + 2y = 10$

(ii) The area of the window (A) expressed as a function of x is

(a) $A = x - \frac{\pi x^3}{8} - \frac{x^2}{2}$ (b) $A = 5x - \frac{x^2}{2} - \frac{\pi x^2}{8}$

(c) $A = 5x - \frac{x^2}{2} - \frac{3x^2}{8}$ (d) $A = 5x + \frac{x^2}{2} + \frac{\pi x^2}{8}$

(iii) Dr. Ritam is interested in maximising the area of the whole window. For this to happen the value of x should be

(a) $\frac{20}{\pi}$ (b) $\frac{20}{4-\pi}$ (c) $\frac{20}{2+\pi}$ (d) $\frac{20}{4+\pi}$

(iv) For maximum value of A, the breadth of rectangular part of window is

(a) $\frac{20}{4+\pi}$ (b) $\frac{20}{\pi}$ (c) $\frac{10}{4+\pi}$ (d) $\frac{5}{2+\pi}$

(v) The maximum area of window is

(a) $\frac{100}{(4+\pi)^2}$ sq.m (b) $\frac{10\pi}{(4+\pi)^2}$ sq.m (c) $\frac{800}{(4+\pi)^2}$ sq.m (d) $\frac{200+50\pi}{(4+\pi)^2}$ sq.m

Sol. (i) Since perimeter of window = $x + y + y$ + perimeter of semicircle

$$= x + 2y + \frac{1}{2} \times 2\pi \times \frac{x}{2} \quad \left[\text{Here radius of semicircle is } \frac{x}{2}\right]$$

$$= x + 2y + \frac{\pi x}{2}$$

Option (c) is correct.

(ii) $A = x \times y + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$

$$= x \times y + \frac{\pi x^2}{8} = x\left(5 - \frac{x}{2} - \frac{\pi x}{4}\right) + \frac{\pi x^2}{8} \quad \left[\begin{array}{l}\because\ 10 = x + 2y + \frac{\pi x}{2} \\ \Rightarrow\ y = \left(5 - \frac{x}{2} - \frac{\pi x}{4}\right)\end{array}\right]$$

$$= 5x - \frac{x^2}{2} - \frac{\pi x^2}{4} + \frac{\pi x^2}{8} = 5x - \frac{x^2}{2} - \frac{\pi x^2}{8}$$

Option (b) is correct.

(*iii*) For maximum value of A

$$\frac{dA}{dx} = 0$$

$$\Rightarrow \quad 5 - x - \frac{\pi x}{4} = 0 \quad \Rightarrow x + \frac{\pi x}{4} = 5$$

$$\Rightarrow \quad 4x + \pi x = 20 \quad \Rightarrow x(4+\pi) = 20$$

$$\Rightarrow \quad x = \frac{20}{4+\pi}$$

Option (*d*) is correct.

(*iv*) $\because y = 5 - \frac{x}{2} - \frac{\pi x}{4}$

$$= 5 - \left(\frac{2x + \pi x}{4}\right) = 5 - x\left(\frac{2+\pi}{4}\right)$$

$$= 5 - \frac{20}{4+\pi} \cdot \frac{2+\pi}{4} = \frac{20(4+\pi) - 20(2+\pi)}{4(4+\pi)}$$

$$= \frac{80 + 20\pi - 40 - 20\pi}{4(4+\pi)} = \frac{40}{4(4+\pi)}$$

$$= \frac{10}{4+\pi}$$

Option (*c*) is correct.

(*v*) Area of window = ar(rectangular part) + ar(semi circular part)

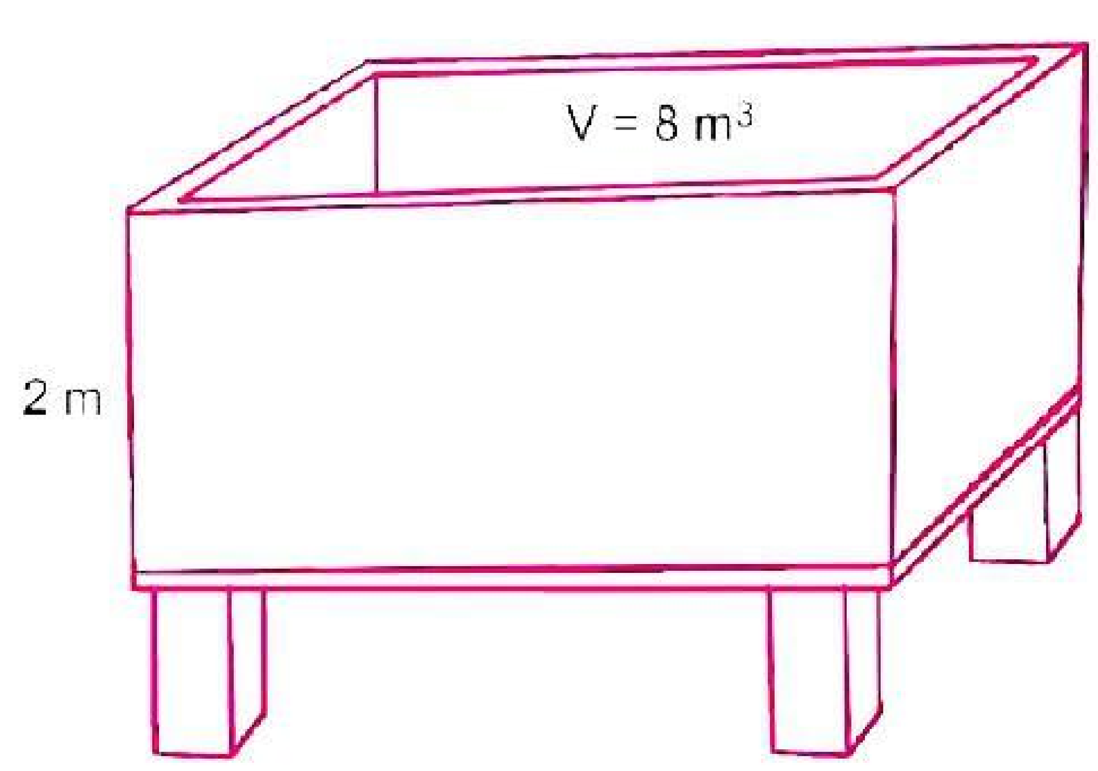
$$= \frac{20}{4+\pi} \cdot \frac{10}{4+\pi} + \frac{1}{2}\pi \cdot \left(\frac{x}{2}\right)^2$$

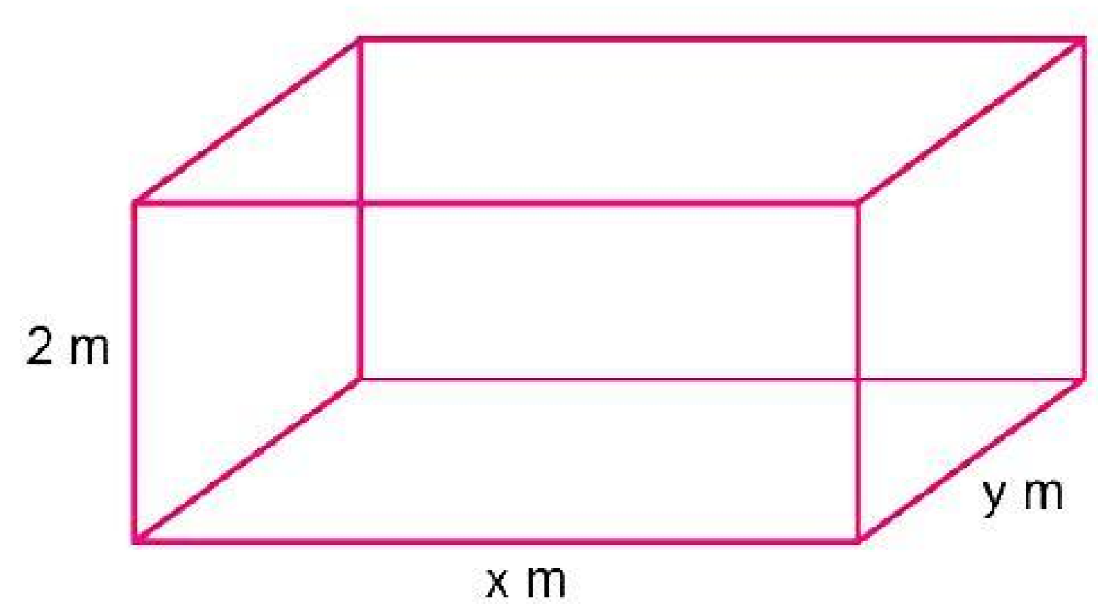
$$= \frac{200}{(4+\pi)^2} + \frac{\pi x^2}{8} = \frac{200}{(4+\pi)^2} + \frac{\pi}{8}\left(\frac{20}{4+\pi}\right)^2$$

$$= \frac{200}{(4+\pi)^2} + \frac{400\pi}{8(4+\pi)^2} = \frac{200 + 50\pi}{(4+\pi)^2}$$

Option (*d*) is correct.

4. On the request of villagers, a construction agency designs a tank with the help of an architect. Tank consists of rectangular base with rectangular sides, open at the top so that its depth is 2 m and volume is 8 m3 as shown below:





Based on the above information answer the following questions:

(*i*) **If x and y represent the length and breadth of its rectangular base, then the relation between the variables is**

(*a*) $x + y = 8$ (*b*) $x \cdot y = 4$ (*c*) $x + y = 4$ (*d*) $\frac{x}{y} = 4$

(*ii*) **If construction of tank cost ₹70 per sq. metre for the base and ₹45 per square metre for sides, then making cost 'C' expressed as a function of x is**

(*a*) $C = 80 + 80\left(x + \frac{4}{x}\right)$ (*b*) $C = 280x + 280\left(x + \frac{4}{x}\right)$

(*c*) $C = 280 + 180\left(x + \frac{4}{x}\right)$ (*d*) $C = 70x + 70\left(x + \frac{x}{4}\right)$

(*iii*) **The owner of a construction agency is interested in minimizing the cost 'C' of whole tank, for this to happen the value of x should be**

(*a*) 4 m (*b*) 3 m (*c*) 1 m (*d*) 2 m

(*iv*) **For minimum cost 'C' the value of y should be**

(*a*) 1 m (*b*) 3 m (*c*) 2 m (*d*) 4 m

(*v*) **The Pradhan of village wants to know minimum cost. The minimum cost is**

(*a*) ₹2000 (*b*) ₹4000 (*c*) ₹11,000 (*d*) ₹1000

Sol. (*i*) Volume of tank = length × breadth × height (Depth)

$$8 = x.y.\,2$$

$\Rightarrow \quad 2xy = 8 \quad \Rightarrow \quad xy = 4$

Option (*b*) is correct.

(*ii*) Since 'C' is cost of making tank

$\therefore \quad C = 70xy + 45 \times 2\,(2x + 2y)$

$\Rightarrow \quad C = 70xy + 90\,(2x + 2y)$

$\Rightarrow \quad C - 70xy + 180\,(x + y)$

$\Rightarrow \quad C = 70x \times \frac{4}{x} + 180\left(x + \frac{4}{x}\right)$

$\Rightarrow \quad C = 280 + 180\left(x + \frac{4}{x}\right)$

$\left[\because\ 2.x.y = 8 \Rightarrow y = \frac{8}{2x} \Rightarrow y = \frac{4}{x}\right]$

Option (*c*) is correct.

(*iii*) For maximum or minimum

$$\frac{dC}{dx} = 0$$

$$\frac{d}{dx}\left(280 + 180\left(x + \frac{4}{x}\right)\right) = 0 \quad \Rightarrow 180\left(1 + 4\left(-\frac{1}{x^2}\right)\right) = 0$$

$\Rightarrow \quad 180\left(1 - \frac{4}{x^2}\right) = 0 \quad \Rightarrow 1 - \frac{4}{x^2} = 0$

$\Rightarrow \quad \frac{4}{x^2} = 1 \quad \Rightarrow x^2 = 4$

$\Rightarrow \quad x = \pm 2$

$\Rightarrow \quad x = 2$ (length can never be negative)

Now, $\frac{d^2C}{dx^2} = 180\left(+\frac{8}{x^3}\right)$

$\Rightarrow \quad \left.\frac{d^2C}{dx^2}\right]_{x=2} = 180 \times \frac{8}{8} = 180 = +\text{ve}$

Hence, to minimize C, $x = 2$m

Option (d) is correct.

(iv) $\because \quad xy = 4$

$\Rightarrow \quad y = \frac{4}{x} \quad \Rightarrow \quad y = \frac{4}{2} \quad \Rightarrow \quad y = 2$ m

Option (c) is correct.

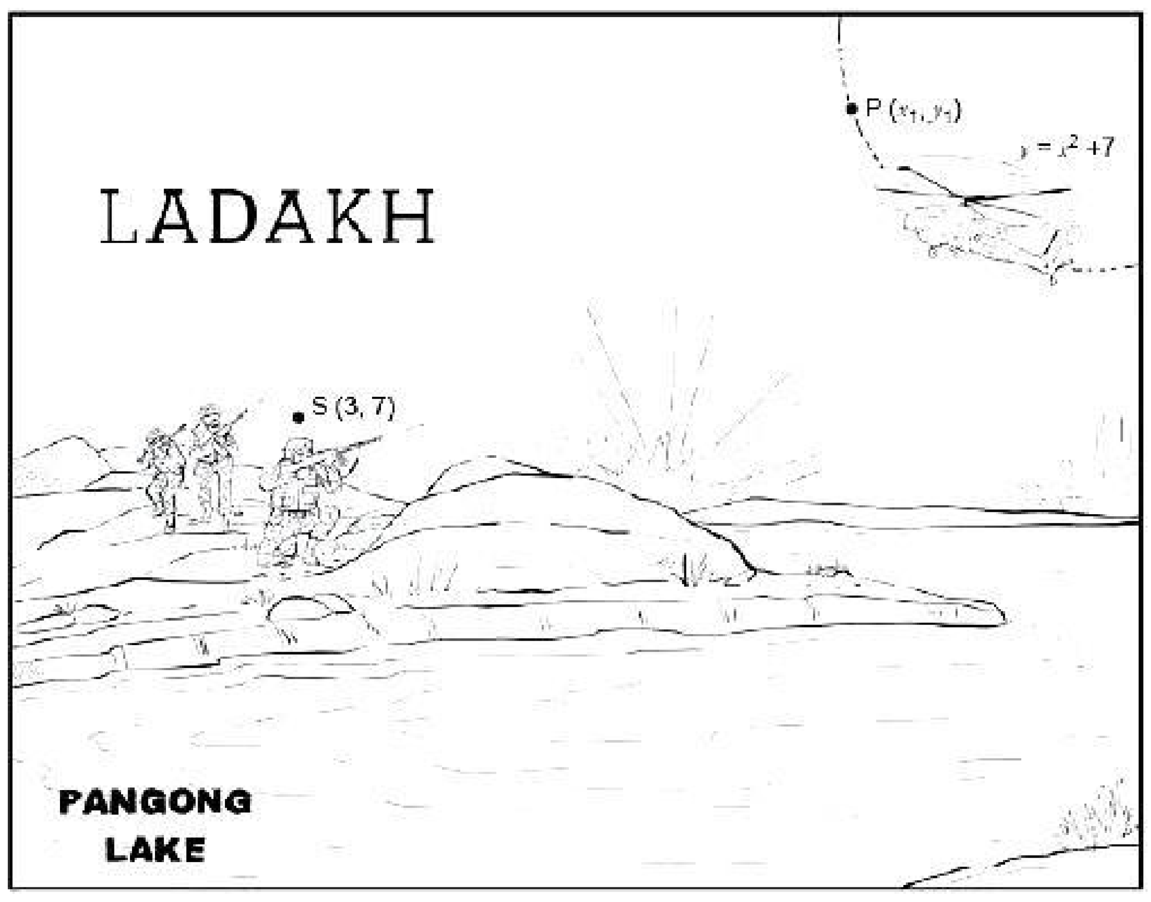
(v) $\because \quad C = 280 + 180\left(x + \frac{4}{x}\right) = 280 + 180\ (2 + 2)$

$= 280 + 180 \times 4 = 280 + 720 =$ ₹1000

Option (d) is correct.

5. These days chinese and Indian troops are engaged in aggressive melee, face-offs skirmishes at locations near the disputed Pangong Lake in Ladakh.

One day a helicopter of enemy is flying along the curve represented by y = x2 +7. A soldier placed at (3, 7) wants to shoot down the helicopter when it is nearest to him.



Based on above information answer the following questions:

(i) If (x_1, y_1) represents the position of helicopter on the curve $y = x^2 + 7$, when the distance D from soldier placed at S(3, 7) is minimum, then the relation between x_1, y_1 is

(a) $x_1 = y_1^2 + 7$ (b) $y_1 = x_1^2 + 7$ (c) $y_1 + x_1^2 = 7$ (d) $y_1^2 + x_1 = 7$

(ii) The distance 'D' expressed as a function of x_1 is

(a) $D = x_1^? - 6x_1 + x_1^4$ (b) $D = x_1^? - 6x_1 + 9 + x_1^4$

(c) $D^2 = x_1^2 - 6x_1 + 9 + x_1^4$ (d) $D^2 = x_1^2 + 6x_1 - 9 + x_1^4$

(iii) The soldier at S wants to know when the enemy helicopter is nearest to soldier, then the value y_1 should be

(a) 4 (b) 3 (c) 8 (d) 5

(iv) When the enemy helicopter is nearest to soldier, then the value of D should be

(a) 4 units (b) 5 units (c) $\sqrt{5}$ units (d) $\sqrt{7}$ units

(v) The nearest position of helicopter from soldier is

(a) $(1, \sqrt{5})$ (b) (1, 8) (c) (1, 7) (d) $(1, \sqrt{7})$

Sol. (i) $\because (x_1, y_1)$ lie on curve $y = x^2 + 7$

$\Rightarrow \quad (x_1, y_1)$ satisfy the equation of curve

$\Rightarrow \quad y_1 = x_1^2 + 7$

Option (b) is correct.

(ii) Here, since soldier is at (3, 7)

$$D = \sqrt{(x_1-3)^2+(y_1-7)^2} \quad \Rightarrow \quad D^2 = (x_1-3)^2+(y_1-7)^2$$

$\because$ (x_1, y_1) lie on curve $y = x^2 + 7$

$\Rightarrow$ $y_1 = x_1^2 + 7$

$\therefore$ $D^2 = (x_1-3)^2 + (x_1^2+7-7)^2$

$D^2 = x_1^2 - 6x_1 + 9 + x_1^4$

Option (*c*) is correct.

(*iii*) We have $D^2 = x_1^2 - 6x_1 + 9 + x_1^4$

$\therefore$ $\dfrac{d(D^2)}{dx_1} = 2x_1 - 6 + 4x_1^3$

For minimum value of D *i.e.* D^2

$$\frac{d(D^2)}{dx_1} = 0$$

$\Rightarrow$ $2x_1 + 4x_1^3 - 6 = 0$

$\Rightarrow$ $4x_1^2(x_1-1) + 4x_1(x_1-1) + 6(x_1-1) = 0$

$\Rightarrow$ $(x_1-1)(4x_1^2+4x_1+6) = 0$

$\Rightarrow$ $x_1 - 1 = 0$

[$\because$ $4x_1^2 + 4x_1 + 6 = 0$, have no real roots *i.e*, real value of x_1 is not possible.]

$\Rightarrow$ $x_1 = 1$

$\Rightarrow$ For minimum points, (distance) or nearest distanct $x_1 - 1$

Also $\dfrac{d^2(D^2)}{dx_1^2} = 2 + 12x_1^2 \quad \Rightarrow \left.\dfrac{d^2(D^2)}{dx_1^2}\right]_{x=1} = +ve$

Since (x_1, y_1) lie on curve $y = x^2 + 7$

$\Rightarrow$ $y_1 = x_1^2 + 7$

$\Rightarrow$ $y_1 = 1^2 + 7$ [For nearest value of D, $x_1 = 1$]

$= 1 + 7 = 8$

Option (*c*) is correct.

(*iv*) For nearest distance D *i.e.* minimum value of D, $x_1 = 1$

$\because$ $D^2 = x_1^2 - 6x_1 + 9 + x_1^4$

$\Rightarrow$ $D^2 = 1^2 - 6(1) + 9 + (1)^4$

$\Rightarrow$ $D^2 = 1 - 6 + 9 + 1$

$\Rightarrow$ $D^2 = 5$

$\Rightarrow$ $D = \sqrt{5}$ units

Option (*c*) is correct.

(*v*) For minimum value of D

$x_1 = 1$ and $y_1 = 8$

$\therefore$ Nearest position of helicopter is (1, 8).

Option (*b*) is correct.

ASSERTION-REASON QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(*a*) *Both A and R are true and R is the correct explanation of A.*

(*b*) *Both A and R are true but R is not the correct explanation of A.*

(*c*) *A is true but R is false.*

(*d*) *A is false and R is also false.*

1. **Assertion (A) :** The rate of change of area of a circle with respect to its radius r when $r = 6$ cm is $12\pi\ \text{cm}^2/\text{cm}$.

   Reason (R) : Rate of change of area of a circle with respect to its radius r is $\frac{dA}{dr}$, where A is the area of the circle.

2. **Assertion (A) :** $f(x) = \tan x - x$ always increases.

   Reason (R) : Any function $y = f(x)$ is increasing if $\frac{dy}{dx} > 0$.

3. **Assertion (A) :** $f(x) = x^4$ is decreasing in the interval $(0, \infty)$.

   Reason (R) : Any function $y = f(x)$ is decreasing if $\frac{dy}{dx} < 0$.

4. **Assertion (A) :** The slope of the tangent to the curve $y = x^3$ where it cuts x-axis, is 0.

   Reason (R) : Slope of tangent to the curve $y = f(x)$ at point (x_0, y_0) is $\frac{dy}{dx}$ at (x_0, y_0).

Answers

1. (*a*) **2.** (*a*) **3.** (*d*) **4.** (*a*)

HINTS/SOLUTIONS OF SELECTED MCQS

1. We have, $f(x) = x^2 e^{-x}$

   $\rightarrow f'(x) = -x^2 e^{-x} + 2x e^{-x} = x e^{-x}(2 - x)$

   for $f(x)$ to be strictly increasing, $f'(x) > 0$

   $\Rightarrow x e^{-x}(2 - x) > 0 \Rightarrow x(2 - x) > 0$

   $\Rightarrow x(x - 2) < 0 \Rightarrow 0 < x < 2$

   $\therefore x \in (0, 2)$

   Option (*d*) is correct.

2. Let (x_1, y_1) be the point on the given curve $3y = 6x - 5x^3$ at which the normal passes through the origin. Then we have $\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 2 - 5x_1^2$. Again the equation of the normal at (x_1, y_1) passing through the origin gives $2 - 5x_1^2 = \frac{-x_1}{y_1} = \frac{-3}{6 - 5x_1^2}$. Since $x_1 = 1$ satisfies the equation, therefore, correct answer is (*a*).

3. We have, $f(x) = x^x$

   Let $y = x^x$

and $\log y = x \log x$

$\therefore \quad \frac{1}{y}.\frac{dy}{dx} = x.\frac{1}{x} + \log x.1$ [Differentiate both sides]

$\Rightarrow \quad \frac{dy}{dx} = (1 + \log x).x^x$

$\because \quad \frac{dy}{dx} = 0 \Rightarrow (1 + \log x).\ x^x = 0$ [$\because x^x \neq 0$]

$\Rightarrow \quad \log x = -1 \Rightarrow \log x = \log e^{-1}$

$\Rightarrow \quad x = e^{-1} \Rightarrow x = \frac{1}{e}$

Hence, $f(x)$ has a stationary point at $x = \frac{1}{e}$.

Option (*b*) is correct.

4. From first equation of the curve, we have $3x^2 - 3y^2 - 6xy\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{x^2 - y^2}{2xy} = (m_1)$ say and second equation of the curve gives

$6xy + 3x^2\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0 \quad \Rightarrow \quad \frac{dy}{dx} = \frac{-2xy}{x^2 - y^2} = (m_2)$say

Since $m_1 \cdot m_2 = -1$. Therefore, the cut at right angles

Option (*b*) is correct.

6. $\frac{dy}{dx} = \cos x$. Therefore, slope of normal $= \left(\frac{-1}{\cos x}\right)_{x=0} = -1$.

Hence, the equation of normal is $y - 0 = -1\ (x - 0)$ or $x + y = 0$.

Therefore, correct answer is (*c*).

7. Here, $y = e^{2x}$

$\Rightarrow \quad \frac{dy}{dx} = 2e^{2x} \quad \Rightarrow \quad \left.\frac{dy}{dx}\right]_{(0,\ 1)} = 2 \times e^0 = 2 \times 1 = 2$

$\therefore$ Slope of tangent to the curve $y = e^{2x}$ at $(0, 1) = 2$

$\Rightarrow$ Equation of tangent to the curve $y = e^{2x}$ at $(0, 1)$ is

$\frac{y - 1}{x - 0} = 2 \quad \Rightarrow \quad y = 2x + 1$

For meeting point with x-axis, putting $y = 0$, we get

$0 = 2x + 1 \quad \Rightarrow \quad x = -\frac{1}{2}$

Hence required point is $\left(-\frac{1}{2}, 0\right)$.

Option (*c*) is correct.

11. Let $f(x) = x^{1/x}$

$\therefore \quad f'(x) = x^{1/x}\left(\frac{1 - \log x}{x^2}\right)$

For critical point

$f'(x) = 0 \quad \Rightarrow \quad 1 - \log x = 0$

$\Rightarrow \quad \log x = 1 \quad \Rightarrow \quad x = e$ (critical point)

Now, $f''(x)]_{x=e} = -$ ve

$\Rightarrow \quad x = e$ is maxima of $f(x) = x^{1/x}$.

$\Rightarrow \quad e^{1/e}$ is greatest value of $f(x) = x^{1/x}$.

Also, $f(4) = 4^{1/4} = 2^{2 \times 1/4} = 2^{1/2} = f(2)$

[$\because$ By Rolle's theorem $f(a) = f(b)$ $\therefore \exists\, c\,(a, b)$ s.t. $f'(c) = 0$]

And $1 < 2 < e < 3 < 4 < 5 < 6 < 7$

Hence, $3^{1/3}$ is the greatest number.

Option (*b*) is correct.

13. Given, $y = x^2 + ax + 25 \qquad \Rightarrow \qquad \frac{dy}{dx} = 2x + a \qquad \ldots(i)$

The curve (*i*) touches the *x*-axis implies that *x* – axis is tangent to curve at meeting point.

$\Rightarrow \quad \frac{dy}{dx} = 0 \qquad \Rightarrow \quad 2x + a = 0$

$\Rightarrow \quad x = -\frac{a}{2}$

$\Rightarrow$ The co-ordinate of meeting point are $\left(-\frac{a}{2}, 0\right)$, therefore it satisfies the curve (*i*)

$\Rightarrow \quad \left(-\frac{a}{2}\right)^2 + a\left(-\frac{a}{2}\right) + 25 = 0$

$\Rightarrow \quad \frac{a^2}{4} - \frac{a^2}{2} + 25 = 0 \qquad \Rightarrow \qquad -a^2 + 100 = 0$

$\Rightarrow \quad a = \pm 10$

Option (*b*) is correct.

16. $f(x) = 2x + \cos x$

$f'(x) = 2 - \sin x > 0 \; \forall x \in R$

$\Rightarrow \quad f(x)$ is an increasing function

Option (*c*) is correct.

17. $f(x) = 2 \sin 3x + 3 \cos 3x$

$\Rightarrow \quad f'(x) = 6 \cos 3x - 9 \sin 3x$

$\Rightarrow \quad f''(x) = -18 \sin 3x - 27 \cos 3x$

$\therefore \quad f''\left(\frac{5\pi}{6}\right) = -18 \sin\left(\frac{5\pi}{2}\right) - 27 \cos\left(\frac{5\pi}{2}\right)$

$\Rightarrow \quad f''\left(\frac{5\pi}{6}\right) = -18 \sin\left(2\pi + \frac{\pi}{2}\right) - 27 \cos\left(2\pi + \frac{\pi}{2}\right)$

$= -18 \sin \frac{\pi}{2} - 27 \cos \frac{\pi}{2} = -18 < 0$

$\therefore$ At $x = \frac{5\pi}{6}$, $f(x)$ is minimum.

Option (*b*) is correct.

18. $y = x^{1/2} \Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$\therefore \quad \left.\frac{dy}{dx}\right|_{(0,0)} = \infty$

$\Rightarrow$ Tangent of the curve $y = x^{1/2}$ at $(0, 0)$ is parallel to *y*-axis.

Option (*b*) is correct.

21. $f(x) = \sin x - ax + b$

$\Rightarrow \quad f'(x) = \cos x - a$

For increasing function

$f'(x) \geq 0$

$\cos x - a \geq 0 \Rightarrow \cos x \geq a$

i.e. $a \leq \cos x \qquad a \leq \min(\cos x) = -1$

$\therefore \quad a \in (-\infty, -1)$.

So option (*c*) is correct.

22. $f(x) = \dfrac{2x^2 - 1}{x^4}$

$f'(x) = \dfrac{x^4(4x) - (2x^2-1)(4x^3)}{(x^4)^2} = \dfrac{4x^5 - 8x^5 + 4x^3}{x^8}$

$= \dfrac{-4x^5 + 4x^3}{x^8} = \dfrac{-4x^2 + 4}{x^5}$

f is decreasing if $f'(x) \leq 0$

$\Rightarrow \quad \dfrac{-4x^2+4}{x^5} \leq 0 \Rightarrow \dfrac{4x^2 - 4}{x^5} \geq 0$

$\Rightarrow \quad x^2 - 1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow |x| \geq 1$ and $x > 0$

$\Rightarrow \quad x \in [1, \infty)$

Option (*b*) is correct.

25. We have $e^y = 1 + x^2$

$\Rightarrow \quad e^y \dfrac{dy}{dx} = 2x$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{2x}{e^y} = \dfrac{2x}{1+x^2} \qquad [\because e^y = 1 + x^2]$

$\Rightarrow \quad m = \dfrac{2x}{1+x^2}$ or $|m| = \dfrac{2|x|}{1+|x|^2}$

As $1 + |x|^2 - 2|x| = (1 - |x|)^2 \geq 0$

$\Rightarrow \quad 1 + |x|^2 \geq 2|x| \qquad \Rightarrow \qquad 1 \geq \dfrac{2|x|}{1+|x|^2} = |m|$

$\Rightarrow \quad |m| \leq 1$

Option (*d*) is correct.

26. We have

$y = x^3 - ax^2 + x + 1 \Rightarrow \dfrac{dy}{dx} = 3x^2 - 2ax + 1$

From question $\dfrac{dy}{dx} \geq 0 \Rightarrow 3x^2 - 2ax + 1 \geq 0 \; \forall \; x$

$\therefore D \leq 0 \Rightarrow 4a^2 - 12 \leq 0 \Rightarrow 4a^2 \leq 12 \Rightarrow a^2 \leq 3$

$\Rightarrow |a| \leq \sqrt{3} \Rightarrow -\sqrt{3} \leq a \leq \sqrt{3}$

Option (*c*) is correct.

27. We have $y = x^2 - x \Rightarrow \dfrac{dy}{dx} = 2x - 1$

Slope of tangent $m = \frac{dy}{dx} = 2x - 1$ ...(*i*)

Since the line $y = 2$ cuts the curve $y = x^2 - x$

$\Rightarrow \quad 2 = x^2 - x \Rightarrow x^2 - x - 2 = 0$

$\Rightarrow \quad x^2 + x - 2x - 2 = 0 \Rightarrow x(x+1) - 2(x+1) = 0$

$\Rightarrow \quad (x+1)(x-2) = 0$

$\Rightarrow \quad x = -1$ or 2

Point of intersection of the line $y = 2$ and the curve

$y = x^2 - x$ are $(-1, 2)$, $(2, 2)$

As point $(2, 2)$ lies in first quadrant

$\therefore$ Slope of tangent at $(2, 2)$ from (*i*) is $m = 2 \times 2 - 1 = 3$

Option (*b*) is correct.

29. $y = \cos^{-1}(\cos x)$

$\Rightarrow \quad \cos y = \cos x$

$\Rightarrow \quad -\sin y \frac{dy}{dx} = -\sin x$

$\Rightarrow \quad \frac{dy}{dx} = \frac{\sin x}{\sin y}$

$$\left.\frac{dy}{dx}\right|_{x=-\frac{\pi}{4}} = \frac{\sin\left(-\frac{\pi}{4}\right)}{\sin\left(-\frac{\pi}{4}\right)} = \frac{-\frac{1}{\sqrt{2}}}{-1/\sqrt{2}} = 1$$

Option (*a*) is correct.

32. $S = b^2x + c^2y$ and $xy = a^2$

$\Rightarrow \quad S = b^2x + \frac{a^2c^2}{x}$

$\therefore \quad \frac{dS}{dx} = b^2 - \frac{c^2a^2}{x^2} = 0$

$\rightarrow \quad b^2 - \frac{c^2a^2}{x^2} = 0 \rightarrow x^2 = \frac{c^2a^2}{b^2} \rightarrow x = \pm\frac{ca}{b}$

$$\frac{d^2S}{dx^2} = \frac{2c^2a^2}{x^3}$$

$$\therefore \quad \left.\frac{d^2S}{dx^2}\right|_{x=\frac{ca}{b}} = \frac{2c^2a^2}{\frac{c^3a^3}{b^3}} = \frac{2b^3}{ca} > 0$$

$\therefore$ Minimum value of $S = b^2 \times \frac{ca}{b} + \frac{c^2a^2}{ca/b} = abc + abc = 2abc$

Option (*c*) is correct.

33. $f(x) = x^4 - x^2 - 2x + 6$

$\Rightarrow \quad f'(x) = 4x^3 - 2x - 2$

$\therefore \quad f'(x) = 0 \Rightarrow 4x^3 - 2x - 2 = 0$

$\Rightarrow \quad 2x^3 - x - 1 = 0$

$\Rightarrow \quad 2x^3 - 2x^2 + 2x^2 - x - 1 = 0$

$\Rightarrow \quad 2x^3 - 2x^2 + 2x^2 - 2x + x - 1 = 0$

$\Rightarrow \quad 2x^2(x-1) + 2x(x-1) + 1(x-1) = 0$

$\Rightarrow \quad (x-1)(2x^2 + 2x + 1) = 0$

$\Rightarrow \quad x = 1$ as $2x^2 + 2x + 1 \neq 0$ for any real x.

$\therefore \quad f''(x) = 12x^2 - 2$

$\therefore \quad f''(1) = 12 - 2 = 10 > 0$

Global minimum value of $f(x) = 1^4 - 1^2 - 2 \times 1 + 6 = 4$.

Option (c) is correct.

36. $f(x) = \cos x + \cos(\sqrt{2}x)$

$\therefore \quad f(x) = 2\cos\frac{\sqrt{2}+1}{2}x \cos\frac{\sqrt{2}-1}{2}x \leq 2$

and it is 2 when $\cos\frac{\sqrt{2}+1}{2}x$ and $\cos\frac{\sqrt{2}-1}{2}$ are both equal to 1 for a value of x. This is possible only when $x = 0$.

Option (b) is correct.

39. Let $f(x) = |x|$ is not differential

$\because f'(0) \neq 0$ but $f(x)$ has a minimum at $x = 0$

Option (d) is correct.

40. $f(x) = \sin^4 x + \cos^4 x \Rightarrow f'(x) = 4\sin^3 x \cos x - 4\cos^3 x \sin x$

$f(x)$ is increasing if $f'(x) > 0$

$\Rightarrow 4\sin^3 x \cos x - 4\cos^3 x \sin x > 0$

$\Rightarrow 4\sin x \cos x(\sin^2 x - \cos^2 x) > 0$

$\Rightarrow -2(2\sin x \cos x)(\cos^2 x - \sin^2 x) > 0$

$\Rightarrow -2\sin 2x \cos 2x > 0$

$\Rightarrow -\sin 4x > 0$

$\therefore -\sin 4x > 0 \qquad \Rightarrow \sin 4x < 0$

$\Rightarrow \pi < 4x < 2\pi \qquad \Rightarrow \frac{\pi}{4} < x < \frac{\pi}{2}$

$\Rightarrow \frac{\pi}{4} < x < \frac{3\pi}{8}$

Option (b) is correct.

41. $f'(x) = \frac{\sin x - x\cos x}{\sin^2 x}, g'(x) = \frac{\tan x - x\sec^2 x}{\tan^2 x}$

Now $\frac{d}{dx}(\sin x - x\cos x) = \cos x + x\sin x - \cos x$

$= x\sin x > 0$ for $0 \leq x < 1$

$\therefore \quad \sin x - x\cos x$ is an increasing function.

But at $x = 0$, $x \sin x$ is 0

$\therefore$ In $0 < x \leq 1, \sin x - x\cos x > 0$

$\therefore \quad f'(x) > 0$ for $0 < x \leq 1$

So $f(x)$ is increasing in the interval $0 < x \leq 1$.

Again $\frac{d}{dx}(\tan x - x\sec^2 x) = \sec^2 x - 2x\sec^2 x \tan x - \sec^2 x$

$= -2x\sec^2 x \tan x < 0$ for $0 \leq x \leq 1$

$\therefore g(x)$ is decreasing in $0 < x \le 1$

Option (*c*) is correct.

42. $h(x) = f(x) - \{f(x)\}^2 + \{f(x)\}^3$

$h'(x) = f'(x) - 2(f(x)f'(x) + 3\{(f(x)\}^2 f'(x)$

$= \{1 - 2(f(x) + 3\{(f(x)\}^2\}f'(x)$

$= 3\left\{\frac{1}{3} - \frac{2}{3}f(x) + (f(x))^2\right\}f'(x)$

$= \left(3\left[(f(x))^2 - \frac{2}{3}f(x) + \frac{1}{9}\right] + \frac{2}{3}\right)f'(x)$

$= \left[3\left\{f(x) - \frac{1}{3}\right\}^2 + \frac{2}{3}\right]f'(x)$

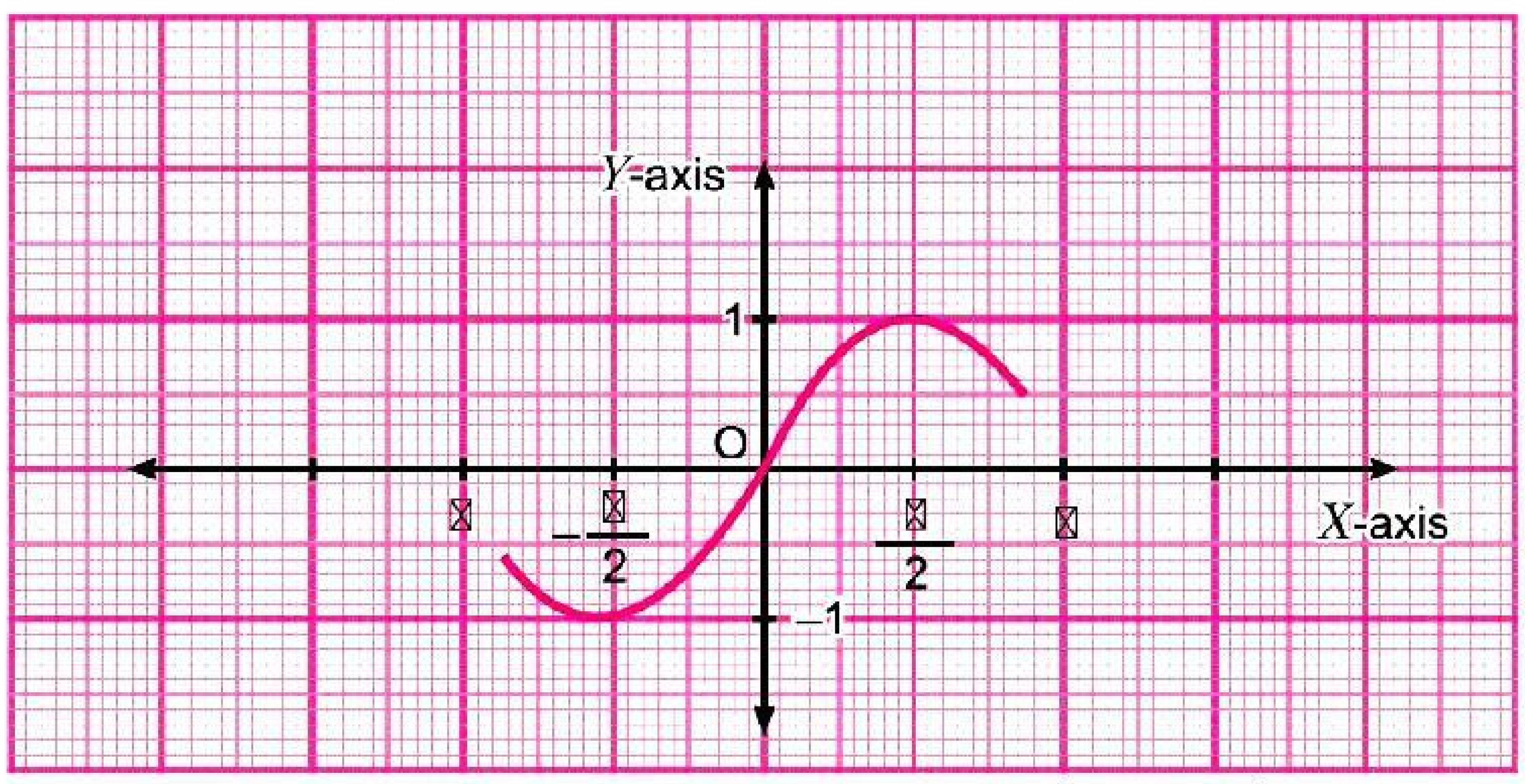
If $f(x)$ is increasing, $f'(x) > 0$ and therefore $h'(x) > 0$

i.e., $h(x)$ is increasing

If $f(x)$ is decreasing, $f'(x) < 0$ and $h'(x) < 0$ *i.e.*, $h(x)$ is decreasing.

Option (*a*) is correct.

43. $f(x) = \sin x$



It is clear from the graph that $f(x)$ is increasing in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$f(f(x)) = \sin(\sin x)$

$\therefore \frac{d}{dx}\{f(f(x))\} = \cos(\sin x)\cos x \ge 0, -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$\therefore f\{f(x)\}$ is increasing in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

As $f\{f(x)\}$ is strictly increasing in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $f\left(f\left(-\frac{\pi}{2}\right)\right) = \sin\left(\sin\left(-\frac{\pi}{2}\right)\right)$

$= \sin(-1) = -\sin 1$

and $f\left(f\left(\frac{\pi}{2}\right)\right) = \left(\sin\left(\sin\frac{\pi}{2}\right)\right) = \sin 1$

and the values $\pm\sin 1$ are not attained at any point in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, we find that $f(f(x))$ is invertible in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Option (*d*) is correct.

44. $f(x) = 2\sin^3 x - 3\sin^2 x + 12\sin x + 5$

$f'(x) = 6\sin^2 x\cos x - 6\sin x\cos x + 12\cos x$

$= 6\cos x\,\{\sin^2 x - \sin x + 2\}$

$= 6\cos x\left\{\sin^2 x - 2\sin x \times \frac{1}{2} + \frac{1}{4} - \frac{1}{4} + 2\right\}$

$= 6\cos x\left\{\left(\sin x - \frac{1}{2}\right)^2 + \frac{7}{4}\right\} \geq 0 \;\forall\; x \in \left[0, \frac{\pi}{2}\right]$

$\therefore f(x)$ is increasing in $\left[0, \frac{\pi}{2}\right]$

Option (b) is correct.

46. $x = e^t \cos t, \frac{dx}{dt} = e^t \cos t - e^t \sin t = e^t(\cos t - \sin t)$

$y = e^t \sin t, \frac{dy}{dt} = e^t \sin t + e^t \cos t = e^t(\sin t + \cos t)$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos t + \sin t}{\cos t - \sin t}$

$\therefore$ Slope of tangent to curve at $\frac{\pi}{4} = \infty$

Tangent makes an angle of $\frac{\pi}{2}$ with the x-axis.

Option (d) is correct.

49. $f(x) = 2x^3 + 9x^2 + 12x - 1$

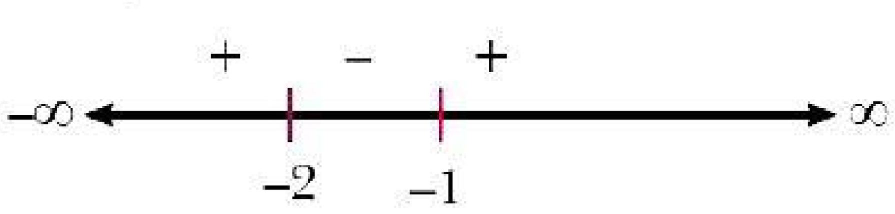
$f'(x) = 6x^2 + 18x + 12$

f is decreasing so $f'(x) \leq 0$

i.e., $6x^2 + 18x + 12 \leq 0 \Rightarrow x^2 + 3x + 2 \leq 0$

$\Rightarrow x^2 + x + 2x + 2 \leq 0 \Rightarrow x(x+1) + 2(x+1) \leq 0$

$\Rightarrow (x+1)(x+2) \leq 0$



$\therefore f(x)$ is decreasing in $[-2, -1]$.

Option (b) is correct.

50. $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$

$\Rightarrow f'(x) = 12\sin^2 x\cos x - 12\sin x\cos x + 12\cos x$

$\Rightarrow f'(x) < 0$ if $x \in \left(\frac{\pi}{2}, \pi\right)$

Option (b) is correct.

51. $f(x) = \tan x - x$

$\Rightarrow f'(x) = \sec^2 x - 1 \geq 0$

$\because$ Range $(\sec x) = (-\infty, -1] \cup [1, \infty)$, hence $f(x)$ always increases.

Option (a) is correct.

52. $f(x) = x^2 - 8x + 17$

$\Rightarrow f'(x) = 2x - 8 = 0 \Rightarrow x = 4$

$f''(x) = 2 > 0$

$x = 4$ point of minima

Minimum value of $f(x) = 16 - 32 + 17 = 1$

Option (c) is correct.

53. $f(x) = x^3 - 18x^7 + 96x$

$f(x) \geq 0 \ \forall \ x \in [0, 9]$

$\Rightarrow$ Minimuim value of $f(x)$ in $[0, 9]$ is 0.

Option (b) is correct.

55. $f(x) = \sin x \cos x = \frac{1}{2} \sin 2x$

$\because -1 \leq \sin 2x \leq 1.$

$\therefore -\frac{1}{2} \leq \frac{1}{2} \sin 2x \leq \frac{1}{2}$

Hence maximum value of f is $\frac{1}{2}$.

Option (b) is correct.

56. $y = -x^3 + 3x^2 + 9x - 27$

$y'(x) = -3x^2 + 6x + 9 =$ slope of the curve $= m$

$\therefore \quad m'(x) = -6x + 6 = 0 \Rightarrow x = 1$

$m''(x) = -6$ and $m''(1) = -6 < 0$

Hence $x = 1$ is the point where slope is minimum.

Hence minimum value of slope

$m(1) = -3 + 6 + 9 = 12$

Option (b) is correct.

57. We have $x + y = 8 \Rightarrow y = 8 - x$

Let $f(x) = xy = x(8 - x) = 8x - x^2$

$\Rightarrow f'(x) = 8 - 2x = 0 \Rightarrow x = 4$

$\Rightarrow y = 4$

$f''(x) = -2$

$f''(4) = -2 < 0$

$\therefore f(x)$ is minimum when $x = 4$

$\therefore$ Maximum value of $f(x) = xy = 16$

Option (b) is correct.

59. Given curve $y = 2x^2 - 2x + 7$

$\therefore$ Slope of tangent $= \frac{dy}{dx} = 4x - 2$

$\therefore$ Slope of normal $= -\frac{1}{4x - 2}$

Given $-\frac{1}{4x-2} = \frac{1}{6} \Rightarrow -6 = 4x - 2$

$\Rightarrow 4x = -4 \Rightarrow x = -1$

When $x = -1$, $y = 2 + 2 + 7 = 11$

$\therefore$ Required point $= (-1, 11)$

Option (c) is correct.

62. We are given curve $2y^2 = ax^2 + b$ ... (A)

and point (1, – 1) is $2 = a + b, a + b = 2$ ...(*i*)

Differentiating (A), w.r.t x we get

$$4y\frac{dy}{dx} = 2ax \Rightarrow \frac{dy}{dx} = \frac{ax}{2y}$$

$$\therefore \frac{dy}{dx}\bigg|_{(1,-1)} = -\frac{a}{2} \quad \text{Given } \frac{dy}{dx}\bigg|_{(1,-1)} = -1$$

$\Rightarrow a = 2$

(*i*) $\Rightarrow b = 2 - 2 = 0$

$\therefore a = 2, b = 0$

Option (*a*) is correct.

64. $f(x) = x + \sin 2x$

$f(0) = 0$ and $f(2\pi) = 2\pi$

Hence $f(x)$ has maximum value 2π and minimum value is 0.

Option (*a*) is correct.

65. $R(x) = 5x^3 - 4x^2, R'(x) = 15x^2 - 8x$

$R'(x)|_{x=20} = 6000 - 160 = 5840$

Option (*b*) is correct.

68. $y = 12x - x^2 \Rightarrow \frac{dy}{dx} = 12 - 2x$

$\because$ Slope of tangent = 0

$\Rightarrow 12 - 2x = 0 \Rightarrow x = 6$

$\therefore x = 6 \Rightarrow y = 72 - 36 = 36$

$\therefore$ Required point is (6, 36).

Option (*c*) is correct.

69. $f(x) = -x^2 - 2x + 15$

$f'(x) = -2x - 2 = -2(x + 1) > 0$

if $x < -1$ *i.e.*, in $(-\infty, -1)$

$f'(x) < 0$ if $x > -1$ *i.e.*, in $(-1, \infty)$

Hence $f(x)$ is increasing in $(-\infty, -1)$ and decreasing in $(-1, \infty)$

Option (*a*) is correct.

71. $f'(x) = 4x^3 - 62 \times 2x + a$

$f'(x) = 4x^3 - 124x + a$

As function attains maximum at $x = 1 \in [0, 2]$

$$f'(1) = 0$$

$\Rightarrow 4 - 124 + a = 0 \Rightarrow a = 120$

Option (*c*) is correct.